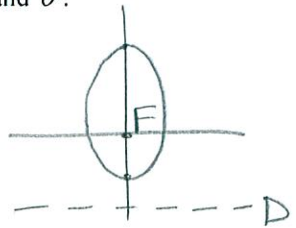
An ellipse has a focus at the pole and vertices with **rectangular** co-ordinates $(0, -5)$ and $(0, 8)$. **SCORE: _____ / 20 PTS**

[a] Give polar co-ordinates for the vertices, using positive values of r and θ.



$\left(5, \frac{3\pi}{2}\right)$ 2 $\left(8, \frac{\pi}{2}\right)$ 2

[b] Find the **polar** equation of the ellipse.

$$r = \frac{ep}{1 - e\sin\theta}$$

$5 = \frac{ep}{1+e}$ 2 $8 = \frac{ep}{1-e}$ 2

$ep = 5 + 5e = 8 - 8e$ 1

$13e = 3$

$e = \frac{3}{13}$ 1

$\frac{3}{13}p = 5 + \frac{15}{13}$ 1

$3p = 65 + 15$

$p = \frac{80}{3}$ 1

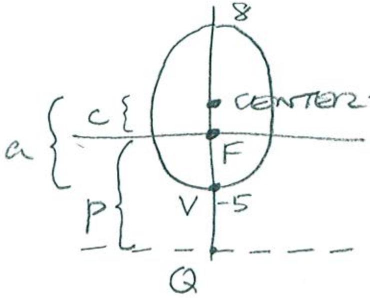
$$r = \frac{\boxed{\frac{80}{13}}}{1 - \frac{3}{13}\sin\theta}$$ 1 1

$$r = \frac{80}{13 - 3\sin\theta}$$

2 2

2 SIMPLIFIED

[b] Find the **polar** equation of the ellipse.



CENTER $= \left(0, \frac{8+-5}{2}\right) = \left(0, \frac{3}{2}\right)$

$c = \frac{3}{2}$ 2

$a = \frac{3}{2} - -5 = \frac{13}{2}$ 2

$e = \frac{c}{a} = \frac{3}{13}$ 2

ALTERNATE SOLUTION

$e = \frac{VF}{VQ}$

$\frac{3}{13} = \frac{5}{p-5}$ 1

$3p - 15 = 65$

$p = \frac{80}{3}$ 1

$r = \frac{ep}{1 - e\sin\theta}$

$r = \frac{\frac{80}{13}}{1 - \frac{3}{13}\sin\theta}$ 1 1

$r = \frac{80}{13 - 3\sin\theta}$ 2 2

2 SIMPLIFIED FINAL ANSWER

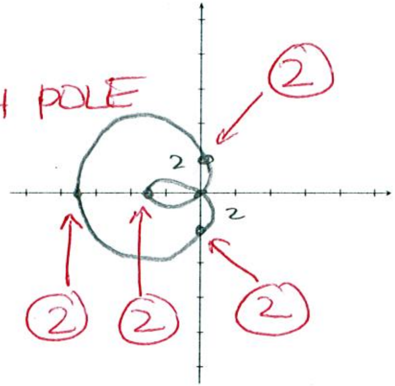
Consider the polar equation $r = 2 - 5\cos\theta$. **SCORE: _____ / 30 PTS**

[a] Sketch the graph of the polar equation using the shortcut process shown in lecture / PowerPoint.
Label the scale on your axes clearly.

θ	r	(x,y)
0	-3	$(-3,0)$
$\pi/2$	2	$(0,2)$
π	7	$(-7,0)$
$3\pi/2$	2	$(0,-2)$

5 SHAPE

2 GOES THROUGH POLE



[b] Convert the equation to rectangular.

$r^2 = 2r - 5r\cos\theta$ 3

3 $x^2+y^2 = 2\sqrt{x^2+y^2} - 5x$ 3

$x^2+y^2+5x = 2\sqrt{x^2+y^2}$ 3

$(x^2+y^2+5x)^2 = 4x^2+4y^2$ 3

OR

$r = 2 - \frac{5x}{r}$ 3

$r^2 = 2r - 5x$ 3

3 $x^2+y^2 = 2\sqrt{x^2+y^2} - 5x$

$x^2+y^2+5x = 2\sqrt{x^2+y^2}$ 3

$(x^2+y^2+5x)^2 = 4x^2+4y^2$ 3

Consider the polar equation $r = 2 - \sin 2\theta$. **SCORE: _____ / 20 PTS**

The following symmetry tests do NOT indicate that the graph is symmetric:
$(-r, -\theta)$, $(-r, \pi-\theta)$ and $(r, \pi-\theta)$

$\theta = \pi/2$ POLAR AXIS $\theta = \pi/2$

[a] **Using the results above, along with the tests and shortcuts shown in lecture**, determine if the graph is symmetric over the polar axis, $\theta = \frac{\pi}{2}$ and/or the pole. Summarize your conclusions in the table on the right.

NOTE: Run as FEW tests as needed to prove your conclusions are correct.

POLAR AXIS: $(r, -\theta)$ $r = 2 - \sin 2(-\theta)$ 2
$r = 2 + \sin 2\theta$ 2

POLE: $(-r, \theta)$ $-r = 2 - \sin 2\theta$ 2
$r = -2 + \sin 2\theta$ 2

$(r, \pi+\theta)$ $r = 2 - \sin 2(\pi+\theta)$ 2
$r = 2 - \sin(2\pi + 2\theta)$
$r = 2 - \sin 2\theta$ 2

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

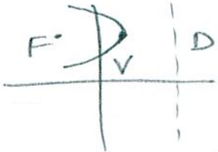
3

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in [0, \pi]$ OR $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ 5

Find the **rectangular** equation of the parabola with focus $(-5, 7)$ and directrix $x = 13$.

SCORE: _____ / 15 PTS



$$\text{VERTEX} = \left(\frac{-5+13}{2}, 7\right) = (4, 7)$$ 3

$p =$ DIRECTED DISTANCE FROM VERTEX TO FOCUS $= -5 - 4 = -9$

4 3 3

$$(y-7)^2 = 4(-9)(x-4)$$

$$(y-7)^2 = -36(x-4)$$

1 1

Name the shapes of the following graphs. **SCORE: _____ / 15 PTS**

[a] the locus of points in the plane that are twice as far from $(7, 7)$ as they are from $x = 7$

HYPERBOLA 3

[b] the locus of points in the plane that are 7 units closer to $(7, -7)$ than they are to $(7, 7)$

HYPERBOLA 3

[c] the graph with polar equation $r = 13 + 7\cos\theta$

LIMAÇON WITH DIMPLE $2\frac{1}{2}$

[d] the graph with polar equation $r = \dfrac{13}{7 - \sin\theta}$

ELLIPSE $2\frac{1}{2}$

[e] the graph with polar equation $\theta = 5$

LINE 2

[f] the graph with equation $7 + 7x^2 - 13y + 13y^2 = 0$

ELLIPSE 2

Consider the polar equation $r = \dfrac{48}{5 - 7\sin\theta}$. **SCORE: _____ / 25 PTS**

[a] Find the equation of the directrix.

$$r = \boxed{\dfrac{\frac{48}{5}}{1 - \frac{7}{5}\sin\theta}} \quad 2$$

$$e = \boxed{\tfrac{7}{5}} \quad 2 \qquad ep = \tfrac{48}{5}$$

$$\boxed{\tfrac{7}{5}p = \tfrac{48}{5}} \quad 1 \longrightarrow p = \tfrac{48}{7}$$

$$\underbrace{y}_{2}\ \underbrace{=}_{2}\ \underbrace{-\tfrac{48}{7}}_{2}$$

[b] Find the **rectangular** coordinates of the endpoints of **all** latera recta. **Do NOT convert the equation to rectangular.**

θ	r	(x, y)
0	$48/5$	$(48/5, 0)$
$\pi/2$	-24	$(0, -24)$
π	$48/5$	$(-48/5, 0)$
$3\pi/2$	4	$(0, -4)$

4

CENTER $= \left(0, \dfrac{-24 + -4}{2}\right) = \underline{(0, -14)}$ 2

FOCI $= (0, 2 * -14)$ AND $(0, 0)$

$= (0, -28), (0, 0)$

ENDS OF L.R. $= (\underbrace{\pm 48/5}_{2}, \underbrace{-28}_{2}), (\underbrace{\pm 48/5}_{2}, \underbrace{0}_{2})$

Consider the conic with rectangular equation $12x^2 - y^2 + 24x + 16y - 16 = 0$. **SCORE: _____ / 25 PTS**

[a] Find the co-ordinates of the focus/foci.

$$12x^2 + 24x - y^2 + 16y = 16$$

$$12(x^2 + 2x) - (y^2 - 16y) = 16$$

$$12(x^2 + 2x + 1) - (y^2 - 16y + 64) = \underbrace{16 + 12 - 64}_{}$$ 3

2 $$\underbrace{12(x+1)^2}_{} - \underbrace{(y-8)^2}_{2} = \underbrace{-36}_{}$$ 1

3 $$\underbrace{\frac{(y-8)^2}{36} - \frac{(x+1)^2}{3} = 1}_{}$$

$$c^2 = 36 + 3 = 39 \longrightarrow c = \underbrace{\sqrt{39}}_{3}$$

$$(\underbrace{-1, 8}_{2} \underbrace{\pm \sqrt{39}}_{2})$$

[b] If the equation corresponds to a circle, find its radius.
If the equation corresponds to a parabola, find its directrix.
If the equation corresponds to an ellipse, find the endpoints of its minor axis.
If the equation corresponds to a hyperbola, find the equations of its asymptotes.

$$\overbrace{y - 8 = \underbrace{\pm}_{} \underbrace{2\sqrt{3}}_{3}(x+1)}^{2}$$ 2